University of New Brunswick
Faculty of Computer Science
CS1303: Discrete Structures
Homework Assignment 3, **Due Time, Date** *11:59 PM, February 23, 2021*

Student Name: ____________________________ Matriculation Number: ________________

Instructor: Rongxing Lu
The marking scheme is shown in the left margin and [100] constitutes full marks.

[**20**] 1. Let $D = \{-48, -14, -8, 0, 1, 3, 16, 23, 26, 32, 36\}$. Determine which of the following statements are true and which are false. Provide counterexamples for the statements that are false.

(a) $\forall x \in D$, if x is odd then $x > 0$.
(b) $\forall x \in D$, if x is less than 0 then x is even.
(c) $\forall x \in D$, if x is even then $x \leq 0$.
(d) $\forall x \in D$, if the ones digit of x is 2, then the tens digit is 3 or 4.
(e) $\forall x \in D$, if the ones digit of x is 6, then the tens digit is 1 or 2.

[**10**] 2. Let $D = E = \{-2, -1, 0, 1, 2\}$. Explain why the following statements are true.

(a) $\forall x$ in D, $\exists y$ in E such that $x + y = 0$.
(b) $\exists x$ in D such that $\forall y$ in E, $x + y = y$.

[**20**] 3. Please rewrite the following statements formally using quantifiers and variables, and write a negation for each statement.

(a) Everybody loves somebody.
(b) Somebody loves everybody.
(c) Any even integer equals twice some integer.
(d) Every action has an equal and opposite reaction.
(e) There is a program that gives the correct answer to every question that is posed to it.

[**50**] 4. Some of the following arguments are valid by universal modus ponens or universal modus tollens; others are invalid. State which are valid and which are invalid. Justify your answers.

(a)

All healthy people eat an apple a day.
Alice eats an apple a day.
$\therefore$ Alice is a healthy person.

(b)

For every student x, if x studies discrete mathematics, then x is good at logic.
Bob studies discrete mathematics.
$\therefore$ Bob is good at logic.

(c)

If compilation of a computer program produces error messages, then the program is not correct.
Compilation of this program does not produce error messages.
$\therefore$ This program is correct.

(d)

Any product of two positive numbers is positive.
The product $p \cdot q$ is positive.
$\therefore$ The numbers p and q are both positive.

(e)

If a number is even, then twice that number is even.
The number $2n$ is even, for a particular number n.
$\therefore$ The particular number n is even.

Solutions.

[**20**] 1. Let $D = \{-48, -14, -8, 0, 1, 3, 16, 23, 26, 32, 36\}$. Determine which of the following statements are true and which are false. Provide counterexamples for the statements that are false.

(a) $\forall x \in D$, if x is odd then $x > 0$.
✓ The statement is true.

(b) $\forall x \in D$, if x is less than 0 then x is even.
✓ The statement is true.

(c) $\forall x \in D$, if x is even then $x \leq 0$.
✓ The statement is false.
Counterexamples:
$16 \in D$ and it is even, but $16 > 0$;
$26 \in D$ and it is even, but $26 > 0$;
$32 \in D$ and it is even, but $32 > 0$;
$36 \in D$ and it is even, but $36 > 0$.

(d) $\forall x \in D$, if the ones digit of x is 2, then the tens digit is 3 or 4.
✓ The statement is true.

(e) $\forall x \in D$, if the ones digit of x is 6, then the tens digit is 1 or 2.
✓ The statement is false.
Counterexamples:
$36 \in D$ and its ones digit is 6, but its tens digit is 3.

[**10**] 2. Let $D = E = \{-2, -1, 0, 1, 2\}$. Explain why the following statements are true.

(a) $\forall x$ in D, $\exists y$ in E such that $x + y = 0$.
✓ The statement is true because for each $x \in D$, its inverse number, denoted as $-x$, is also an element in E, i.e., $-x \in E$. Then, for each $x \in D$, we always have $y = -x \in E$ such that $x + y = x + (-x) = 0$.

x	$y = -x$	$x + y$
-2	2	0
-1	1	0
0	0	0
1	-1	0
2	-2	0

(b) $\exists x$ in D such that $\forall y$ in E, $x + y = y$.
✓ The statement is true because $x = 0 \in D$ is such an element. In specific, for each $y \in E$, we always have $x + y = 0 + y = y$.

$x=0$	y	$x+y=y$
0	-2	-2
0	-1	-1
0	0	0
0	1	1
0	2	2

[**20**] 3. Please rewrite the following statements formally using quantifiers and variables, and write a negation for each statement.

(a) Everybody loves somebody.

✓

- P: domain of all people
- $Love(x, y)$: x loves y
- *Formal version:* $\forall\, x \in P, \exists\, y \in P, Love(x, y)$.
- *Negation:*

$$\begin{aligned} & \neg(\forall\, x \in P, \exists\, y \in P, Love(x, y)) \\ \equiv\ & \exists x \in P, \neg(\exists y \in P, Love(x, y)) \\ \equiv\ & \exists x \in P, \forall y \in P, \neg Love(x, y) \end{aligned}$$

In other words, there is someone who does not love anyone.

(b) Somebody loves everybody.

✓

- P: domain of all people
- $Love(x, y)$: x loves y
- *Formal version:* $\exists x \in P, \forall y \in P, Love(x, y)$.
- *Negation:*

$$\begin{aligned} & \neg(\exists x \in P, \forall y \in P, Love(x, y)) \\ \equiv\ & \forall x \in P, \neg(\forall y \in P, Love(x, y)) \\ \equiv\ & \forall x \in P, \exists y \in P, \neg Love(x, y) \end{aligned}$$

In other words, everyone has someone whom they do not love.

(c) Any even integer equals twice some integer.

✓

- Z: domain of integer
- $Even(x)$: x is an even integer
- $Twice(x, y)$: x is twice of y
- *Formal version:* $\forall x \in Z, Even(x) \rightarrow (\exists y \in Z, Twice(x, y))$.
- *Negation:*

$$\begin{aligned} & \neg(\forall x \in Z, Even(x) \rightarrow (\exists y \in Z, Twice(x, y))) \\ \equiv\ & \exists x \in Z, \neg(Even(x) \rightarrow (\exists y \in Z, Twice(x, y))) \\ \equiv\ & \exists x \in Z, \neg(\neg Even(x) \vee (\exists y \in Z, Twice(x, y))) \\ \equiv\ & \exists x \in Z, Even(x) \wedge \neg(\exists y \in Z, Twice(x, y)) \\ \equiv\ & \exists x \in Z, Even(x) \wedge (\forall y \in Z, \neg(Twice(x, y)) \end{aligned}$$

(d) Every action has an equal and opposite reaction.

✓

- A: domain of actions
- $Equal(x, y)$: x and y are equal
- $Opposite(x, y)$: x and y are opposite
- *Formal version:* $\forall x \in A, \exists y \in A, Equal(x, y) \wedge Opposite(x, y)$.
- *Negation:*

$$\begin{aligned} & \neg(\forall x \in A, \exists y \in A, Equal(x, y) \wedge Opposite(x, y)) \\ \equiv \quad & \exists x \in A, \neg(\exists y \in A, Equal(x, y) \wedge Opposite(x, y)) \\ \equiv \quad & \exists x \in A, \forall y \in A, \neg(Equal(x, y) \wedge Opposite(x, y)) \\ \equiv \quad & \exists x \in A, \forall y \in A, \neg Equal(x, y) \vee \neg Opposite(x, y)) \end{aligned}$$

(e) There is a program that gives the correct answer to every question that is posed to it.

✓

- P: domain of programs
- Q: domain of questions
- $Correct(x, y)$: program x gives the correct answer to question y posed to x.
- *Formal version:* $\exists x \in P, \forall y \in Q, Correct(x, y)$.
- *Negation:*

$$\begin{aligned} & \neg(\exists x \in P, \forall y \in Q, Correct(x, y)) \\ \equiv \quad & \forall x \in P, \neg(\forall y \in Q, Correct(x, y)) \\ \equiv \quad & \forall x \in P, \exists y \in Q, \neg Correct(x, y) \end{aligned}$$

[50] 4. Some of the following arguments are valid by universal modus ponens or universal modus tollens; others are invalid. State which are valid and which are invalid. Justify your answers.

(a)

All healthy people eat an apple a day.
Alice eats an apple a day.
$\therefore$ Alice is a healthy person.

✓ This statement is invalid.

- P: domain of people.
- $Healthy(x)$: x is healthy.
- $Apple(x)$: x eats an apple a day.

Then, we can rewrite the argument as follows,

	All healthy people eat an apple a day.		$\forall x \in P, Healthy(x) \rightarrow Apple(x)$
	Alice eats an apple a day.		$Apple(Alice)$
$\therefore$	Alice is a healthy person.	$\therefore$	$Healthy(Alice)$

Let $x = Alice$ in $\forall x \in P, Healthy(x) \rightarrow Apple(x)$, we can draw a truth table as follows. From the truth table, we can see that, in the critical row (line 3), when $Healthy(Alice) \rightarrow Apple(Alice)$ and $Apple(Alice)$ are true, $Healthy(Alice)$ is false. Thus, the statement is invalid.

	$Healthy(Alice)$	$Apple(Alice)$	$Healthy(Alice) \rightarrow Apple(Alice)$	$Apple(Alice)$	$Healthy(Alice)$
1	T	T	T	T	T
2	T	F	F	F	T
3	F	T	T	T	F
4	F	F	T	F	F

(b)

For every student x, if x studies discrete mathematics, then x is good at logic.
Bob studies discrete mathematics.
$\therefore$ Bob is good at logic.

✓ This statement is valid.

- S: domain of student.
- $Dmath(x)$: x studies discrete mathematics.
- $Logic(x)$: x is good at logic.

Then, we can rewrite the argument as follows,

For every student x, if x studies discrete mathematics, then x is good at logic.	$\forall x \in S, Dmath(x) \rightarrow Logic(x)$
Bob studies discrete mathematics.	$Dmath(Bob)$
$\therefore$ Bob is good at logic.	$\therefore$ $Logic(Bob)$ Universal Modus Ponens

According to the university Modus Ponens, the above statement is valid.

(c)

If compilation of a computer program produces error messages, then the program is not correct.
Compilation of this program does not produce error messages.
$\therefore$ This program is correct.

✓ This statement is invalid.

- P: domain of computer programs.
- $Error(x)$: Compilation of x produces error messages.
- $Correct(x)$: x is correct.

Then, we can rewrite the argument as follows,

If compilation of a computer program produces error messages, then the program is not correct.	$\forall x \in P, Error(x) \rightarrow \neg Correct(x)$
Compilation of this program does not produce error messages.	$\neg Error(\text{Pro})$
$\therefore$ This program is correct.	$\therefore$ $Correct(\text{Pro})$

Let $x =$ this program (denoted by "Pro") in $\forall x \in P, Error(x) \rightarrow \neg Correct(x)$, we can draw a truth table as follows. From the truth table, we can see that, in the critical row (line 4), when $Error(\text{Pro}) \rightarrow \neg Correct(\text{Pro})$ and $\neg Error(\text{Pro})$ are true, $Correct(\text{Pro})$ is false. Thus, the statement is invalid.

	$Error$(Pro)	$Correct$(Pro)	$Error$(Pro) $\rightarrow \neg Correct$(Pro)	$\neg Error$(Pro)	$Correct$(Pro)
1	T	T	T	F	T
2	T	F	F	F	F
3	F	T	T	T	T
4	F	F	T	T	F

(d)

Any product of two positive numbers is positive.
The product $p \cdot q$ is positive.
$\therefore$ The numbers p and q are both positive.

✓ This statement is invalid.

- P: domain of all numbers.
- $Pos(x)$: x is positive
- $PPos(x, y)$: the product of x and y is positive.

Then, we can rewrite the argument as follows,

	Any product of two positive numbers is positive.	$\forall x, y \in P, Pos(x) \wedge Pos(y) \rightarrow PPos(x, y)$
	The product $p \cdot q$ is positive.	$PPos(p, q)$
$\therefore$	The numbers p and q are both positive.	$\therefore$ $Pos(x) \wedge Pos(y)$

Let $x = p$ and $y = q$ in $\forall x, y \in P, Pos(x) \wedge Pos(y) \rightarrow PPos(x, y)$, we can draw a truth table as follows. From the truth table, we can see that, in the critical row (line 4), when $Pos(p) \wedge Pos(q) \rightarrow PPos(p, q)$ and $PPos(p, q)$ are true, $Pos(p) \wedge Pos(q)$ is false. Thus, the statement is invalid.

	$Pos(p)$	$Pos(q)$	$Pos(p) \wedge Pos(q)$	$PPos(p, q)$	$Pos(p) \wedge Pos(q) \rightarrow PPos(p, q)$	$PPos(p, q)$	$Pos(p) \wedge Pos(q)$
1	T	T	T	T	T	T	T
2	T	F	F	F	T	F	F
3	F	T	F	F	T	F	F
4	F	F	F	T	T	T	F

(e)

If a number is even, then twice that number is even.
The number $2n$ is even, for a particular number n.
$\therefore$ The particular number n is even.

✓ This statement is invalid.

- P: domain of all numbers.
- $Even(x)$: x is even.

Then, we can rewrite the argument as follows,

	If a number is even, then twice that number is even.	$\forall x \in P, Even(x) \rightarrow Even(2x)$.
	The number $2n$ is even, for a particular number n.	$Even(2n)$
$\therefore$	The particular number n is even.	$\therefore$ $Even(n)$

Let $x = n$ in $\forall x \in P, Even(x) \rightarrow Even(2x)$, we can draw a truth table as follows. From the truth table, we can see that, in the critical row (line 3), when $Even(n) \rightarrow Even(2n)$ and $Even(2n)$ are true, $Even(n)$ is false. Thus, the statement is invalid.

	$Even(n)$	$Even(2n)$	$Even(n) \to Even(2n)$	$Even(2n)$	$Even(n)$
1	T	T	T	T	T
2	T	F	F	F	T
3	F	T	T	T	F
4	F	F	T	F	F